

**Indian Statistical Institute, Bangalore**

B. Math (Hons.) Third Year

Second Semester - Differential Equations

Back Paper Exam    Duration : 3 hours    Max Marks 50    Date : June 07, 2017

Each question carries 5 marks

1. Find a necessary condition for the equation  $Mdx + Ndy = 0$  to have an integrating factor that is a function of  $z = xy$ .
2. If  $y_1$  and  $y_2$  are independent solutions of the of a second order homogeneous linear differential equation, prove that any solution is of the form  $ay_1 + by_2$  for some constants  $a$  and  $b$ .
3. Solve  $y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x$ .
4. Solve  $(1+x)y' = py$ ,  $y(0) = 1$  and prove  $(1+x)^p = 1 + \sum_{n \geq 1} \frac{p(p-1)\cdots(p-(n-1))}{n!} x^n$  for  $|x| < 1$ .
5. Solve  $(1-x^2)y'' - xy' + p^2y = 0$  near  $x = 1$ .
6. Solve  $x' = x + y$  and  $y' = 4x - 2y$ .
7. Prove Rodrigue's formula.
8. Find solutions  $\Delta u = 0$  for which  $u(x, y) = f(x)g(y)$ .
9. Prove maximum principle for heat equation.
10. State and prove maximum principle for subharmonic functions, that is  $u$  such that  $\Delta u \geq 0$ .